Ampère as a Contemporary Physicist

by R. A. R. TRICKER

The basis of electrodynamics, what Maxwell called its cardinal principle, lies in the laws which govern the action of one current-carrying conductor upon another. These can be expressed in several forms which are more or less equivalent at bottom and capable of being transformed in many cases from one into the other. The earliest of these was the law of action of current elements, according to which the effect of an electric circuit is to be looked upon as the sum of contributions from the current elements of which it is composed. For complete circuits this law can be transformed directly into that of the equivalent magnetic shell and from this the ' work law ' can be derived, if it is preferred to work from that. The name of Ampère is sometimes associated with the law of action of current elements in a general way. Sometimes his name is reserved for a particular expression. Sometimes the names of Laplace, Biot, Savart or Grassman are employed. There is little system about it.

Many of the older books referred to the well known expression

$$H = \frac{i \,.\, ds \,.\, \sin\theta}{r^2} \qquad (1)$$

for the magnetic field H generated by an element ds of a circuit carrying a current i, at a distance r from the element (θ being the angle between the element and the line joining it to the point in question) as Ampère's Law[1]. Occasionally it was called Laplace's Law[2]. In books written more recently it is generally referred to as the law of Biot and Savart. Some call it Grassman's formula.

It was, as most elementary books explain, Oersted who first demonstrated the action of an electric current upon a magnet. He had announced his intention of looking for the effect as early as 1807 but was still demonstrating negative experiments as late as 1819[3]. Others were also engaged in the search and the effect looked for seems to have been the orientation of an electric pile suspended by a thread, or its action on a magnet, but on open circuit. In 1819–20 Oersted tried the effect of closing the circuit but as he held the connecting wire at right angles to the magnet, again he obtained no result. It was at the end of a lecture in which he demonstrated this negative result that he thought of holding the wire parallel to the compass needle and first obtained a positive result. At first it was weak and uncertain but he soon improved upon it and finally announced his finding publicly in July 1820[4]. He did not determine the quantitative laws governing the phenomenon but restricted himself to a qualitative description of his observations.

[1] This is so for example in Jean's *Electricity and Magnetism*, Pidduck's *Treatise on Electricity*, Whetham's *Theory of Electricity*, and Gray's *Dictionary of Physics*.

[2] For example in Hadley's *Magnetism and Electricity for Students*. Philip Parker (*Electricity and Magnetism*) gives it as Ampère's or Laplace's law and states that it was given by both.

[3] See Whittaker, *History of the Theories of Aether and Electricity*, p. 81.

[4] Oersted: Translations in Thomson's *Annals of Philosophy* XVI 1820, p. 273, and in *Journ. of Tel. Engineers*, vol. 5, p. 459, 1876.

After Oersted's break-through results came thick and fast. It is perhaps a pity that his name is not associated with the law to which his observations first gave rise, in addition to those whose contributions were also but limited, but when so many have been concerned it is not possible to commemorate all. Arago, who had just returned from abroad, described Oersted's experiments to a meeting of the French Académie des Sciences on the 11th September 1820. On the 18th September, just a week later, André Marie Ampère (1775-1835)[1] showed that two parallel wires carrying currents attracted each other if the currents flowed in the same direction in both wires and repelled each other if the currents were in opposite directions.

On the 30th October 1820, Biot and Savart[2] announced the results of experiments in which they determined the strength of the magnetic field generated by a current in a long straight wire. They timed the oscillation of a small magnet held at various distances from the wire, the magnetic field of which was arranged sometimes to oppose and sometimes to reinforce that of the earth. Sometimes the earth's field was annulled by means of an auxiliary magnet. This achievement is to be measured in light of the fact that their only source of electric current was the electric pile and elaborate measures had to be taken to allow for its variations. It is remarkable that they succeeded at all under such difficulties. They " were led to the following result, which expresses accurately the actions exerted on a molecule of austral or boreal magnetism " (i.e. a north seeking or south seeking pole) " when placed at any distance from a thin cylindrical wire of indefinite length rendered magnetic by the voltaic current. From the point where the molecule is, draw a perpendicular to the axis of the wire; the force which acts on the molecule is perpendicular to this line and to the axis of the wire. Its intensity varies inversely as the distance ".

On the basis of this result Laplace deduced that the magnetic field produced by an element of current must follow a law of inverse squares. There seems little doubt about this though he does not appear to have written any paper on the subject—at least none appears in his collected papers. The fact is mentioned both by Ampère[3] and Biot[4] himself.

While the experimentation of Biot and Savart had been magnificent, their own theoretical interpretation of the results was much less happy. Speaking of the action of a current element they said:[5] " La nature de son action est la même que celle d'une aiguille aimentée qui serait placée sur le contour du fil[6] dans sens

[1] *Annales de Chimie et de Phys.* Ser. II, XV. p. 59, 1820.

[2] *Annales de Chimie et de Phys.* XV, 1820, p. 222.

[3] *Mém. d'Acad.* 30 Aug. 1826, p. 232.

[4] *Précis Elémentaire de Phys.* II, p. 122. " C'est ce qu'a fait M. Laplace. Il a déduit de nos observations que la loi individuelle des forces élémentaire exercées par chaque tranche du fil conjonctif était la raison inverse du carré de la distance, c'est à dire précisément la même que l'on sait exister dans les actions magnétique ordinaire. Cette analyse montrait que, pour compléter la connaissance de la force il restait encore à déterminer si l'action de chaque tranche du fil était la même dans toutes les directions à distance égale, ou si elle était plus ènergique dans certains sens que dans d'autres."

[5] *Ann. de Chimie et de Phys.* XV, 1820, p. 222.

[6] This is expressed more explicitly in the *Journ. de Phys., de Chim., d'Histoire Naturelle et des Arts* XC, 1820. " La nature de son action est la même que celle d'une aiguille aimentée que serait placée tangentiallement au contour du fil, de sorte qu'une particule de magnétisme austral et une particule de magnétisme boréal en seroient sollicitées en sens contraires, quoique toujours suivant la même droite par la construction précédent."

déterminé et toujours constant par rapport à la direction du courant voltaïque". Ampère[1] pointed out that the field of a small magnetic needle varied inversely as the cube and not the square of the distance and that if the needle was placed tangentially to the cross section of the wire, as it would have to be to produce a field at right angles to the axis of the wire and the perpendicular to the point under consideration, a complete circle of such needles round the circumference of the cross section would form a closed loop producing no external field at all.

Biot went on to measure the magnetic field of a long wire which had been bent at one point. The two pieces (see fig. 1) were held in a vertical plane at equal

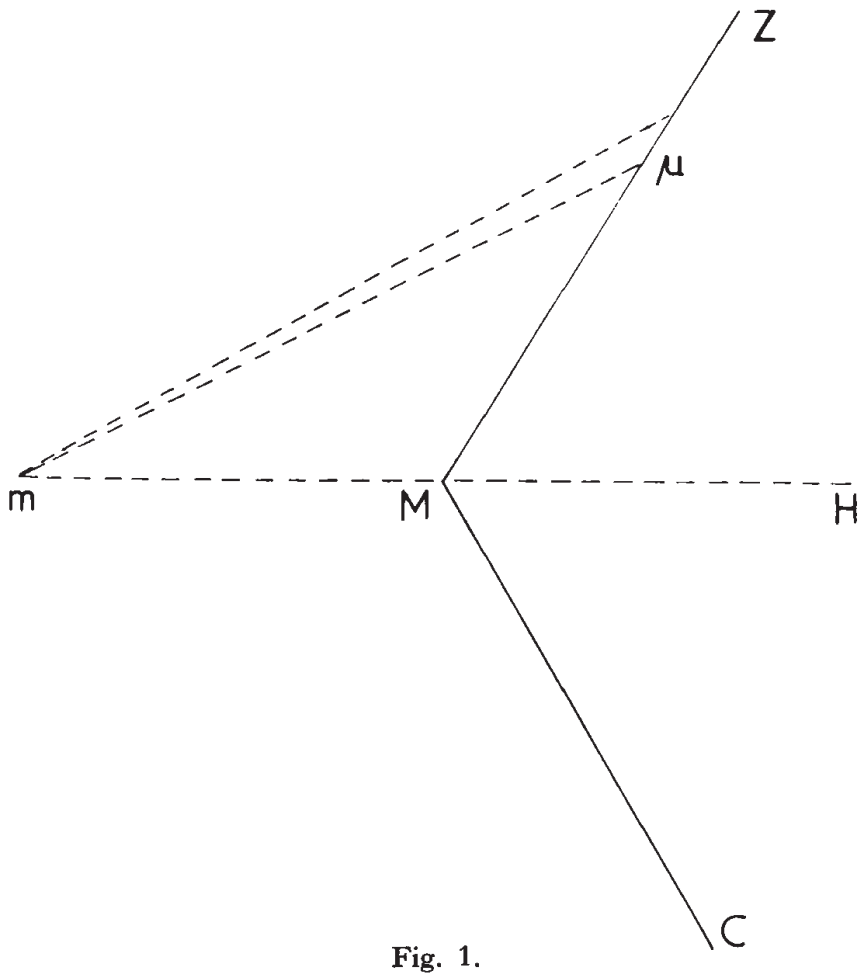

Fig. 1.

angles ZMH, CMH to the horizontal and the forces on a magnetic pole placed at points along the horizontal line mM lying in this plane and passing through the point at which the wire was bent, were measured as before. Biot reported: " J'ai trouvé ainsi que, pour l'un comme l'autre " (i.e. for the bent wire and the long straight wire) " l'action est réciproque à la distance . . . mais l'intensité absolue était plus faible pour le fil oblique que pour le fil droit, dans la proportion de l'angle ZMH à l'unité. Ce résultat, analysé par le calcul, m'a parue indiquer que l'action de chaque élément μ du fil oblique sur chaque molécule m de magnétisme austral ou boréal est réciproque au carré de sa distance μm à cet molécule, et proportionelle au sinus de l'angle mμM formé par la distance μM avec la longuer du fil."[2]

[1] *Mém. d'Acad.*, 1823, p. 357, 360.

[2] *Précis Elémentaire de Phys.*, Tome II, pp. 122 and 123. Quoted by Ampère, *Mém. d'Acad.*, 1823, pp. 281–282.

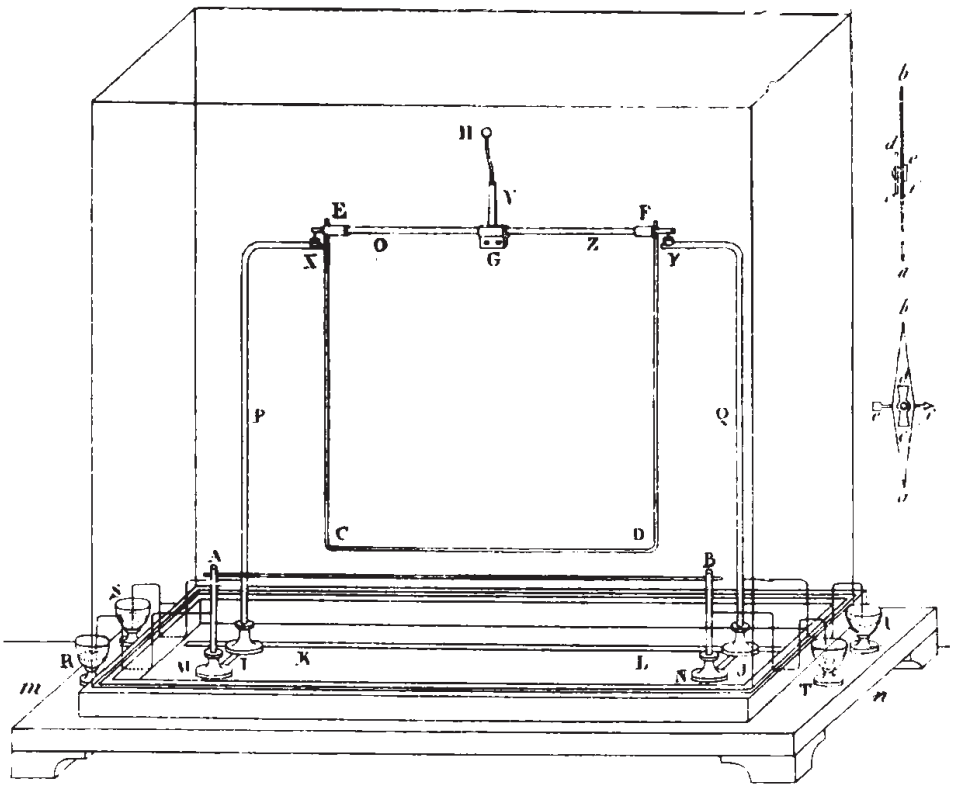

Fig. 2. Ampère's early attempts at finding the law of interaction of current elements employed apparatus in which the force experienced by the circuit carrying a current due to another was directly measured. This is Ampère's diagram of his apparatus for investigating the attraction and repulsion between parallel currents. CD is pivoted at E and F and swings so that it lies in the same horizontal plane as AB along which a second current passes.

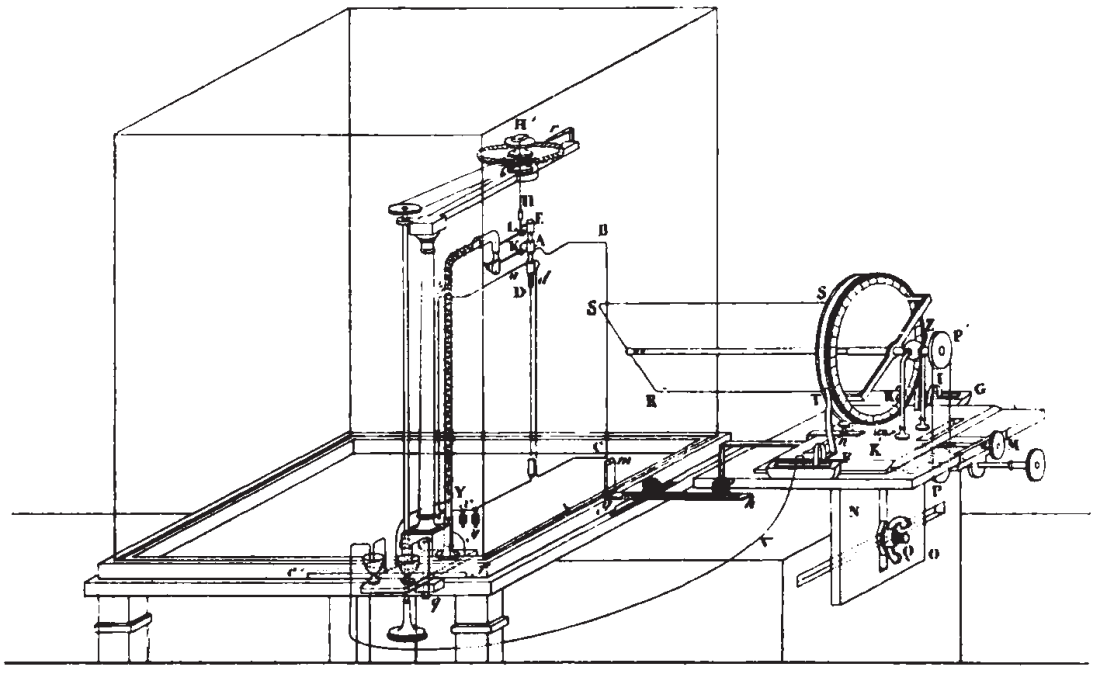

Fig. 3. Ampère's diagram of an apparatus he designed to measure the force exerted by a current in SR upon a current in the suspended wire BC. He soon abandoned this type of investigation because of the difficulty in maintaining steady currents.

Unfortunately there is again an error of calculation. Ampère pointed out that the law which, in fact, is correct, does not follow from the results stated by Biot. To obtain the law it is necessary for the force to vary, not as the angle ZMH but as the tangent of half that angle—as M. Savary had shown (*Mém. d'Acad.* 3rd February 1823). Biot subsequently corrected this mistake.

Ampère's main paper occurs in the *Mémoires de l'Académic de Paris* for 1823. This volume, however, was not published until 1827 and in it, when it was printed, were incorporated later communications so that the order of precedence becomes confused. The date of writing, however—the 30th August 1826—is mentioned in the paper (p. 368). In it are included the results of work already communicated earlier.

Ampère showed himself master of both the experimental and the theoretical situation. His experimentation was brilliant and his theoretical treatment of

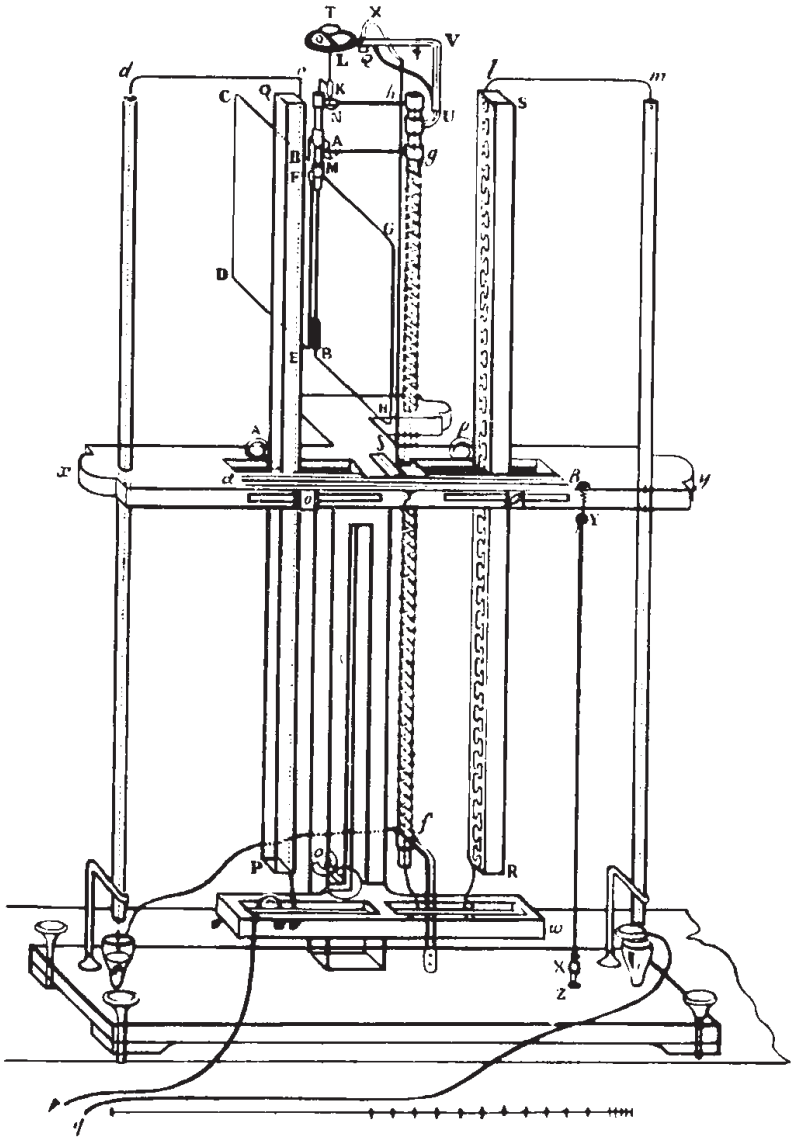

Fig. 4. Ampère's apparatus for showing that a wire could be bent into small zig-zags without affecting the magnetic field it produced at distant points. The wire SR which is bent into zig-zags, is mounted at the same distance from the suspended wire GH as the straight wire PQ. The same current flowing in these two wires produced no deflection in GH. By this experiment Ampère justified his resolution of current elements.

his results masterly. The question which he attacked was this. If the action of one circuit carrying a current upon another is interpreted as the resultant of interactions between pairs of current elements in each circuit, what must be the law of action between them? In his experimental arrangements he soon abandoned attempts to use steady currents in apparatus like that shown in fig. 2 and fig. 3 and designed experiments working on ' null ' principles. His experiments consisted in devising balanced situations in which a suspended circuit was not deflected by a nearby fixed one when current was switched on in both.

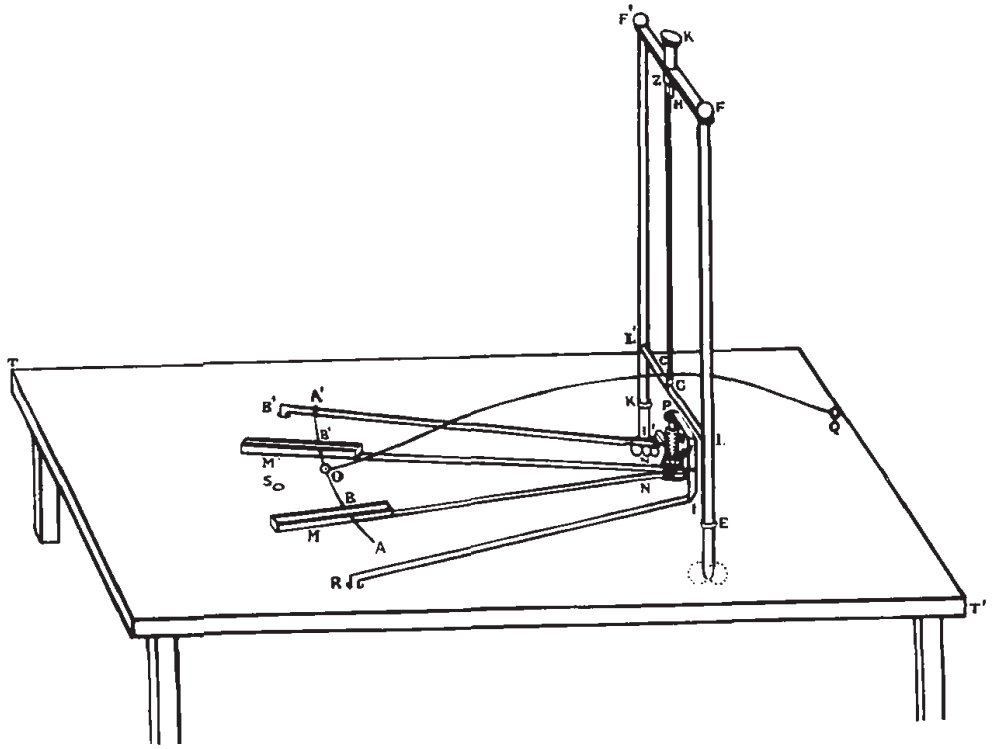

Fig. 5. Ampère's apparatus for showing that the force experienced by a short length of circuit is at right angles to the conductor. The wire AA′ in the form of an arc of a circle is suspended by means of the arm OG and balanced by a counterpoise at Q. A current is sent along AA′ via the mercury cups N and N′ filled brimming full. AA′ experiences no deflection from a second circuit when the supporting arm OG lies along a radius but it does if this is not so.

His theory was based upon four such experiments. The first showed that a current doubledback along itself produced no magnetic field at external points; the second (fig. 4) that a wire bent into the form of small zig-zags produced the same effect as a straight wire at the same distance; the third (fig. 5) that the force on an element of a circuit when acted on by a closed circuit, was always at right angles to the element; the fourth (fig. 6) that if the linear dimensions of a pair of circuits and their distances from one another were multiplied by a constant factor the interaction between them was unaltered. The whole investigation lasted six years and was finally read to the Academy on the 30th August 1826.[1]

In the theoretical field Ampère conceived his problem to be as follows. The third experiment shows that the force upon an element of current exerted

[1] For a simple deduction of the Biot–Savart formula from Ampère's experiments see *On the Teaching of Electricity in Schools* (John Murray).

by another current *forming a complete circuit* is at right angles to the element. Can a law of action between *current elements* be deduced in which the forces each exerts upon the other are equal and opposite and have as their line of action the line joining the elements, thus agreeing with Newton's third law and yet give a force at right angles to one element when integrated round the complete circuit of which the other element forms a part? In this Ampère really restricted himself more than is strictly necessary. To satisfy Newton's law

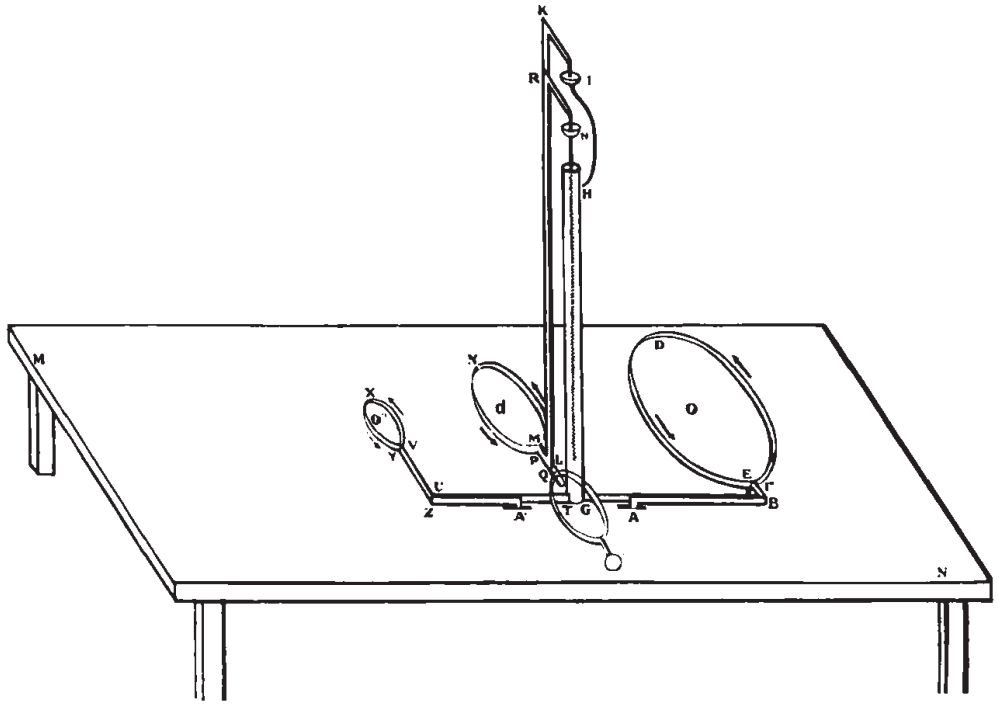

Fig. 6. Ampère's apparatus for showing that the force between two circuits is independent of the linear dimensions as long as the geometry is not otherwise altered. The radii of the three horizontal circles are in geometrical progression. The centre circle is free to move being pivoted by mercury cups I and N. A current sent through the three circles causes no movement of the centre circle if its centre divides the line OO′ in the common ratio of the radii. The force exerted upon the suspended circle by the larger circle at the larger distance is balanced by that exerted by the smaller circle at a correspondingly smaller distance.

it is necessary for the forces to be equal and opposite and to have the same line of action, but it is not necessary that this line of action should pass through the elements themselves.

As early as the 4th December 1820 he announced that the required law must be of the form

$$F=\frac{i\,.\,i'\,.\,ds\,.\,ds'}{r^2}\ (\sin\theta\,.\,\sin\theta'.\,\cos\omega+k\,.\,\cos\theta\cos\theta') \qquad (2)$$

where $i\,.\,ds$ and $i\,.\,ds'$ are the two current elements, r the distance between them, θ and θ' the angles each makes with the line joining them (in the direction of r increasing in each case), ω the angle between the two planes determined by the line joining the elements and the directions of the elements themselves and k is a constant (see fig. 7).

It was not until the great paper of 1826[1] that Ampère gave the value of k to be one half and his formula therefore,

$$F=\frac{i\,.\,i'\,.\,ds\,.\,ds'}{r^2}\,(\sin\theta\,.\,\sin\theta'\,.\,\cos\omega+\tfrac{1}{2}\cos\theta\cos\theta') \qquad (3)$$

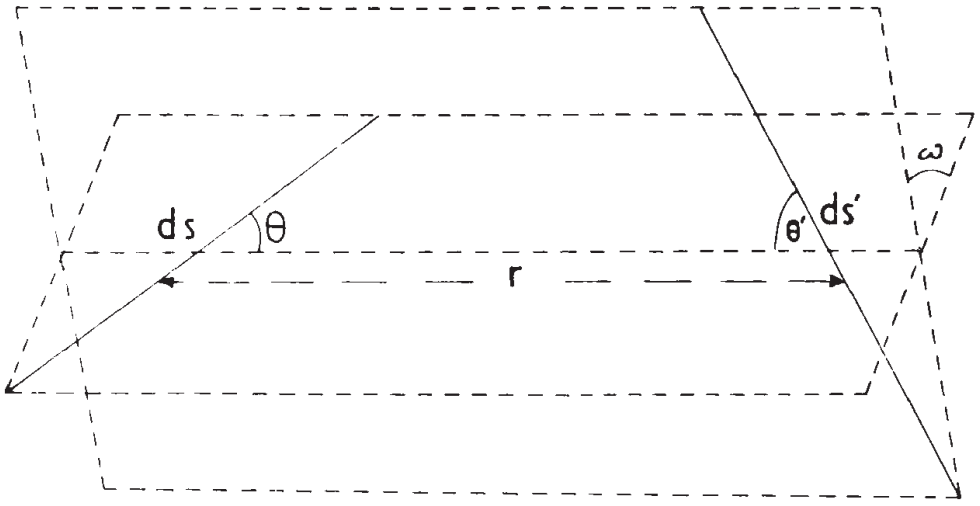

Fig. 7.

In this he employed so-called electrodynamic units which are $\sqrt{2}$ times smaller than electromagnetic units. If, therefore, i and i' are measured in electromagnetic units Ampère's formula becomes

$$F=\frac{i\,.\,i'\,.\,ds\,.\,ds'}{r^2}\,(\cos\theta\,.\,\cos\theta'+2\sin\theta\,.\,\sin\theta'\,.\,\cos\omega) \qquad (4)$$

or as it is more usually expressed,

$$F=\frac{i\,.\,i'\,.\,ds\,.\,ds'}{r^2}\left(\frac{dr}{ds}\,.\,\frac{dr}{ds'}-2r\,.\,\frac{d^2\,.\,r}{ds\,.\,ds'}\right) \qquad (5)$$

The deduction of expression (3) occupied roughly the first half of the paper of 1826. The second half was devoted to a theory of magnetism which ascribed it to circulating molecular currents which, in the aggregate, would be equivalent to a solenoidal current flowing round the magnet. This was shown to account qualitatively for all the phenomena of magnetism and by the law of action of current elements just deduced, also quantitatively for forces between magnets and between them and electric currents.

Ampère used his law of action of current elements to determine the force which a solenoid PQ (fig. 8), infinite in length in one direction (the direction of Q), exerts upon a current element $i\,.\,ds$. He showed that this force was proportional to

$$\frac{i\,.\,ds\,.\,\sin\theta}{r^2}$$

that its line of action passed through ds and was at right angles to the plane containing r and ds. Since this force was the resultant of a large number of forces between ds and the current elements of the solenoid, which themselves experienced equal and opposite forces, it followed that the force acting upon the solenoid due to ds was equal and opposite to that acting upon ds itself and had the same line of action. In other words the force due to ds acting alone would possess a moment about the end of the solenoid.

[1] *Mém. de l'Acad.*, 1826.

Ampère then proceeded to integrate his expression round the circuit of which *ds* formed a part and showed that for a complete circuit the resultant force exerted upon the solenoid passed through the end of the solenoid, which thus behaved as one end of an infinitely long magnet. By considering two such solenoids with currents circulating in opposite directions and displaced relatively to one another he was able to work out similarly the behaviour of a magnet of finite length when placed near an electric current. In this way he established the equivalence of his law of action of current elements with the action between a current and a magnet as investigated by Biot and Savart and others. Remembering that Ampère had no more to go upon than Newton's laws in mechanics, Oersted's qualitative observations and his own experimental results his achievement is remarkable in the extreme. It is small wonder that Maxwell referred to him as the Newton of electricity.

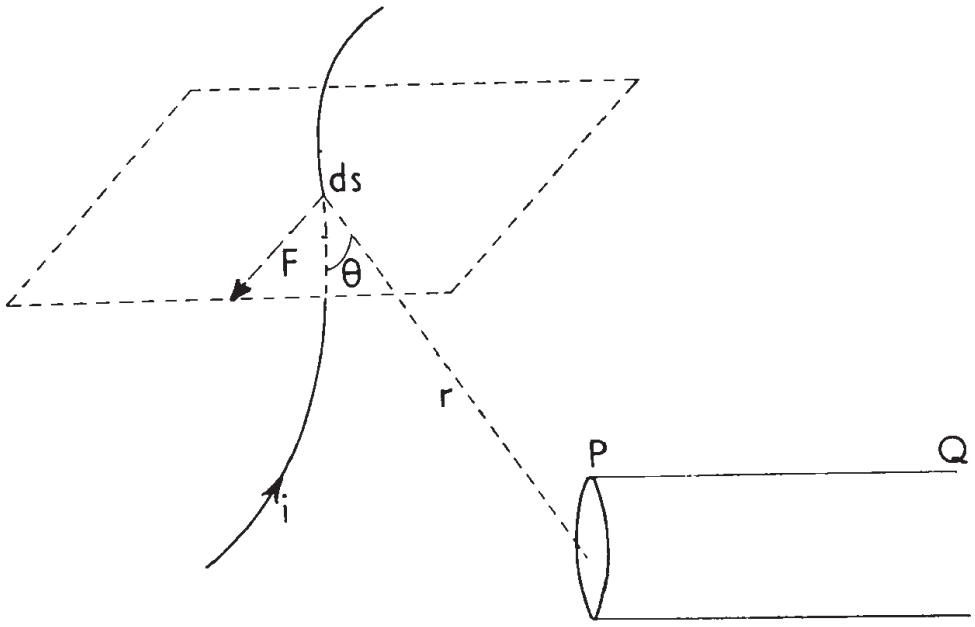

Fig. 8.

Ampère's interpretation of the interaction of two electric circuits as being the sum of the interactions in pairs of the constituent current elements, each member acting upon the other with equal and opposite forces having as their line of action the line joining the elements has since been criticized by many writers. These criticisms have continued down to the present day. What has been criticized has not been so much the idea of current elements, though that has not been altogether exempt, as the particular law which Ampère formulated. It is interesting that such a scientific controversy should have lasted for so long that it brings Ampère into the field of contemporary physics. This is related to the fact that while it is possible to render a small part of a circuit movable and thus measure the force exerted upon it, it is not possible to isolate an element of the other circuit which gives rise to this force and determine its contribution to the total. It has therefore not been possible to subject Ampère's expression to the test of experiment. It is, of course, easy to test the result of the integration of the expression round a complete circuit, at least in certain cases, but many different expressions can be framed which yield the same result on integration

and experiment cannot distinguish between them. The reader may well incline to the view that the discussion of a question which cannot even in principle be put to the test of experiment cannot be looked upon as scientific. Nevertheless this has not prevented Ampère being subjected to criticism by many people.

Among the first to do so was Grassman.[1] Reverting to eqn. (4) and employing ϵ, the angle between ds and ds' themselves in place of ω the angle between the planes defined by r and ds and ds' respectively Grassman put Ampère's formula into the form:—

$$F=\frac{i\,.\,i'\,.\,ds\,.\,ds'}{r^2}\,(2\cos\epsilon+3\cos\theta\,.\,\cos\theta') \tag{6}$$

since

$$\cos\epsilon=\sin\theta\sin\theta'\cos\omega-\cos\theta\cos\theta'. \tag{7}$$

Grassman then made two points, one of which was valid but inconclusive and the other really metaphysical in nature. His first point was that current elements, unlike material particles, possess direction as well as magnitude. There is thus no *a priori* reason why the law of action to which they conform should be the same as that obeyed by material particles. This is, of course, perfectly true and, indeed, would still be true had current elements magnitude only since they are certainly not material particles and might well differ from them in their behaviour however convenient it might be if they turned out to obey the same rules.

Grassman's second point was as follows. Consider the case of two current elements which are parallel so that $\epsilon=0$ and $\theta=\pi-\theta'$ (remembering that θ and θ' were both measured in the direction of r increasing).

Equation (7) then gives us

$$F=\frac{i\,.\,i'\,.\,ds\,.\,ds'}{r^2}\,(2-3\cos^2\theta). \tag{8}$$

If now $\cos^2\theta=2/3$ or $\cos 2\theta=1/3$ the force F becomes zero. Moreover as θ passes through the value $\frac{1}{2}\cos^{-1}1/3$ the force changes from an attraction to a repulsion. Grassman finds this to be so improbable that he rejects it. He does not do so on account of any disagreement with observation and, indeed, no experimental test is possible. His decision is really one of taste. *De gustibus nil disputandum.*

Grassman's difficulty here is that he is not content with a statement that electric currents react on each other *as if* each element of one circuit reacted with each element of the other according to a certain rule. In fact, of course, it is never possible to say more than this. Science can never be more definite about concepts which transcend direct perception by the senses than to say that a situation is as if it were composed in a certain way. This applies to the structure of matter looked upon as consisting of atoms, electrons etc. just as much as to current elements. Grassman was not alone, of course, in his time in wanting to know what was the ' real ', as opposed to a possible explanation.

Grassman's endeavour was to replace the current elements of Ampère, which it was impossible to subject to experiment, by something else which could be so tested. To do this he introduced what he called a ' winkelstrom ', that is, a current flowing along the arms of an angle. He pointed out that a current

[1] Poggendorf's *Annalen*, LXIV, pp. 1–18.

flowing along the sides of the triangle *abc* (fig. 9) could be looked upon as being composed of the three winkelstroms *fabd*, *dbce* and *ecaf*, those arms of the angles lying beyond the triangle, such as *fa* carrying equal and opposite currents from two winkelstroms. Any circuit could thus be resolved into its component winkelstroms which could therefore replace Ampère's current elements. Grassman does not discuss the closure of his winkelstroms but since he envisages the field of a ' winkelstrom ' being tested by experiment he clearly is thinking of it being completed at infinity. He does not, however, base his conclusions on the results of experiments with winkelstroms but uses Ampère's formula to calculate the force they would exert on an element of a second circuit. Since in the case of his winkelstroms he is now considering in effect a complete circuit it is, perhaps, not altogether surprising that he should find the force to be at right angles to the element and in magnitude the same as is given by the formula of Biot and Savart.

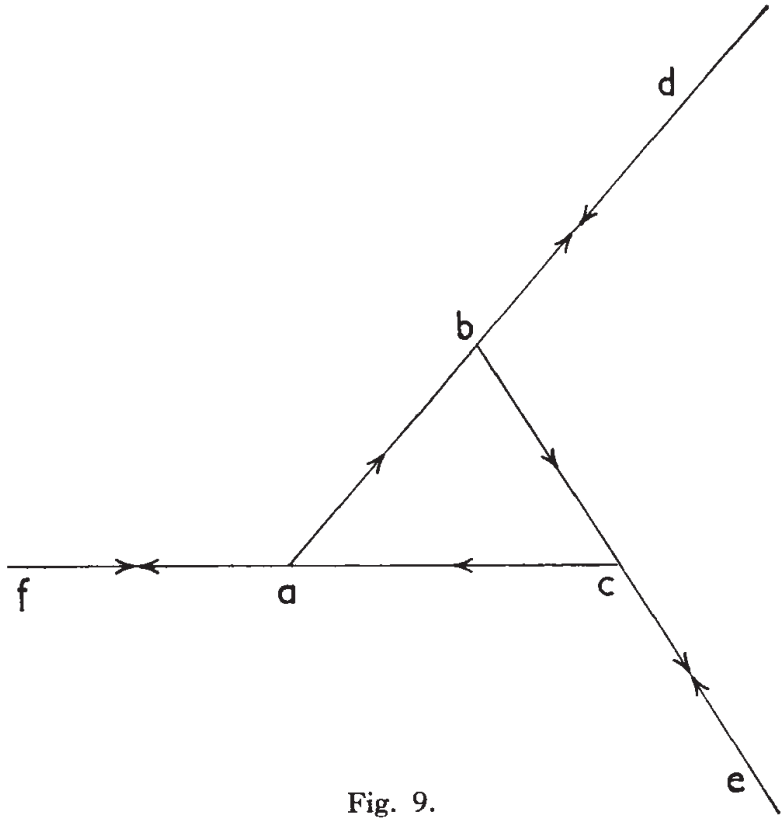

Fig. 9.

A later questioner of Ampère's work was Heaviside. In 1888 he wrote : " It has been stated on no less an authority than that of the great Maxwell, that Ampère's law of force between a pair of current elements is the cardinal formula of electrodynamics. If so, should we not always be using it ? Do we *ever* use it ? Did Maxwell in his treatise ? Surely there is some mistake[1] ". Later in 1893 he wrote:[2] " In fact, in the long series of investigations in electrodynamics to which Oersted's discovery and the work of Ampère, Henry and Faraday gave rise, it was customary to consider an element of a conduction current as generating a certain field of magnetic force. Natural as this course may have seemed, it was an unfortunate one, for it left the question of the closure of the current open; and it is quite easy to see now that this alone constitutes a great

[1] See Whittaker, *op. cit.*, p. 88.
[2] Oliver Heaviside, *Electromagnetic Theory*, vol. I, p. 64.

hindrance to progress. But so far as closed currents are concerned in a medium of uniform inductivity, this way of regarding the relation between current and magnetic force gives equivalent results to those obtained from the first law of circulation " (i.e. $\int H \,.\, ds = 4\pi$ times current enclosed) " in the limited form suitable to the circumstances stated . . . .

But from the Maxwellian point of view this field of H is that corresponding to a certain circuital distribution of electric current, of which the current element is a part; this complete current being related to the current element in the same way as the induction of an elementary magnet is to the intensity of magnetization of the latter. Calling the complete system of electric current a rational current element it may easily be seen that in a circuital distribution of rational current elements the external portion of the current disappears by mutual cancelling and there is left only the circuital current made up of the elements in the older sense. We may therefore employ the formula " (the Biot–Savart formula) " to calculate, without ambiguity, the magnetic force of any circuital distribution of current."

Alas, in spite of its provocative christening the ' rational ' current element proved no more popular than Ampère's formula. Do we, in fact, ever use it ? Nevertheless Heaviside was correct in calling attention to the necessity for considering only complete circuits if the Biot–Savart form of the law is to be employed and that with an isolated current element of the old type the circuit was not closed. If Maxwell's laws and the idea of the displacement current are accepted it follows that an isolated element of steady current cannot exist by itself.

$$\text{curl } \mathbf{H} = \frac{1}{c}(\mathbf{j} + \dot{\mathbf{D}})$$

Since the divergence of any curl is zero it follows that any distribution of moving charges together with their associated displacement currents is non-divergent, that is to say continuous. A current element, in other words, can never be isolated. There is nothing in this, however, which is incompatible with Ampère's conclusions.

In place of Ampère's expression Heaviside wished to emphasize " the law expressing the mechanical force on an element of a conductor supporting current in any magnetic field—the vector product of current and induction ". He wished to attach Ampère's name to it. This still leaves open the question of the generation of the magnetic field by the current which gives rise to it and we still have to face the difficulty of the properties to be ascribed to the element of that circuit. Heaviside solves this problem by his ' rational current element ' which he shows produces a magnetic induction given by the Biot–Savart formula: $i \,.\, ds \,.\, \sin\theta / r^2$.

More recently Whittaker has continued the criticism of Ampère. He says :[1] " The weakness of Ampère's work evidently lies in the assumption that the force is directed along the line joining the elements; for in the analogous case of the action between two magnetic molecules we know that the force is not directed along the line joining the two molecules ". The point is very much the same as that raised by Grassman. It must be repeated, however, that there is not the slightest reason *a priori* why the same law should apply in the two cases. In fact a variety of expressions can be constructed which when integrated round

[1] Whittaker, *History of the Theories of Aether and Electricity*, p. 86.

a circuit agree with experience and it is only in this integrated form that any of them can be tested. What Whittaker was pointing out was that Ampère's solution based upon the assumption that the interaction has for its line of action the line joining the elements is not the most general one possible. In fact it discounts the possibility of there being a turning effect in addition to that tending to purely translational motion. In the case of the magnetic molecules the forces they each exert on the other are equal and opposite and possess the same line of action but this line of action is not generally that joining the molecules. They tend to turn as well as to move each other with a motion of pure translation.

Whittaker rejected the Biot–Savart and Grassman formulae on the grounds that they were not in accordance with Newton's third law. To this extent Ampère would agree with him. However, Whittaker also rejected Ampère's solution as well, on the grounds that it is not the most general and developed another of his own which included the possibility of a turning effect. On integrating round a complete circuit, however, all these formulae yield the same result. Ampère would, no doubt, defend himself from this criticism by pointing out that no experiment performed so far necessitates anything more than a purely translational force to be assumed. His assumption is the simplest to be made. It is in accordance with Newton's laws of motion and with the facts of experience. More than this cannot be asked of any theory.

Ampère, in fact, devoted considerable space to the consideration of the difference between a current element and a magnetic molecule. Much of what he wrote, it is true, is concerned with a discussion of the circumstances under which the force between two bodies can be considered ' truly elementary ' and this being a matter of taste and definition, was not very profitable. Nevertheless, in elaborating his theory of magnetism he showed very clearly that his law of action was capable of explaining quantitatively the action of electric currents on magnets and the action of one magnet on another, covering in so doing the case cited by Whittaker of the interaction of two magnetic molecules.

An objection of a different kind has recently been made by Geoffrey Builder[1]. He begins by mentioning Whittaker's criticisms of Ampère discussed above and also his rejection of the Grassman formula on the grounds that it is inconsistent with Newton's third law. Builder then claims to show that " the Grassman formula is correct and that the Ampère formula is untenable because it is incompatible with the restricted theory of relativity and with the accepted electrodynamical theory of electrons ".

Although his theory is interesting and instructive it does not seem to me that he has substantiated his claim. He starts with the Coulomb force between two charges q and q', q being at the origin of coordinates and q' at a distant point moving with velocity v'. The charge q is then given a velocity v and the resulting force between q and q' calculated by means of the transformation formulae of relativistic mechanics. This is then applied to the moving charges in a current element. Builder says: " A current element $i \,.\, ds$, of a metallic conducting circuit carrying a current i may be represented by n pairs of equal and opposite point charges q/n and $-q/n$ moving with colinear speeds u and v respectively in the direction of the unit vector t tangent to the current element." It is here that the difficulty arises.

[1] Geoffrey Builder, *Bulletin of the Institute of Physics*, vol. 9, 12, 1958.

A current element in an electric circuit carrying a steady current is in a state of dynamic equilibrium. It is connected to a source of electric charge at one end and to a sink at the other. This is no longer true in Builder's model which is equivalent not only to a flow of electric charge but involves a changing electric dipole in addition, produced as his charges separate. The properties of a body may be profoundly modified by its connection to others. My head, for example, is a very different thing (or so I would hope) in its normal position attached to a living body than it would be if severed from it, or to give a non-biological example, I can sit in a railway carriage until I perish from cold or die of starvation but I shall never get to London if it does not form part of a train. When Builder's current elements are connected together to form a circuit the changing dipole moments will cancel each other out. We might therefore expect that the law which he deduces as a result of his calculations will apply to completed circuits but will tell us nothing definite about the action of the individual elements. He has, in fact, provided us with another alternative derivation of the Biot–Savart formula subject to the same limitations as the previous ones. It is, of course well known that his formula gives the correct result for a complete circuit. It in no way invalidates that of Ampère, however.

Rosser has recently given in greater detail in this Journal much the same calculation as Builder, with the suggestion that a deduction of the Biot–Savart formula from the principles of electrostatics and the theory of relativity furnishes the best ground we have for accepting it. It is desirable to examine this suggestion. It amounts to accepting the Biot–Savart formula on the grounds that it occurs in an axiomatic deductive system and the force of the argument will depend upon what other *independent* evidence there is for the acceptance of this system. Any axiomatic system depends for its acceptability upon the direct evidence which exists for each of its deductions and the object of building such a system is to allow the direct evidence for any one deduction to serve as indirect evidence for all the others. If the law of Biot and Savart is included in such a system for which there is other independent evidence, then clearly it gains in strength and Rosser's calculations contribute to its acceptability. It is equally clear, however, that since the acceptability of the system as a whole depends upon the direct evidence for each of the constituent deductions, the calculation cannot by itself entirely displace the basis of the law in experiment. Moreover since the mechanics of moving charges furnishes a very important part of the evidence supporting the special theory of relativity, without which the theory would be in a very much less assured position than it is, Rosser's argument cannot be pushed very far. We cannot use the mechanics of moving charges to establish the theory of relativity and then use the theory of relativity to establish that of mechanics. Furthermore to do what is required the deduction would have to establish the law to an accuracy of something like 1 in 10^5—the accuracy of the current balance. On the face of it it seems unlikely that a theory dependent upon the approximate evaluation of second order effects could achieve this.

Moving packets of charge, such as those contemplated by Builder and Rosser, do not produce steady fields. Associated with the fields which they carry with them is a distribution of energy which is propagated through space. This introduces a third entity into the considerations. If energy and momentum are transferred to and from these fields the forces which two such packets experience due

to each other need certainly not be equal and opposite. The difficulty, if difficulty it be, disappears under steady conditions. This kind of explanation was considered by Ampère when he discussed the possibility of forces arising through a reaction from the ether. His conclusion was, " Sans qu'on soit autorisé à rejeter les explications fondées sur la réaction de l'éther mis en mouvement par les courants électriques, rien n'oblige jusqu'à présent d'y avoir recours."[1]

So long as complete circuits are being considered it is only a matter of convenience whether Ampère's formula or that of Biot and Savart is employed. The latter is almost always simpler to use and, as Heaviside remarked, Ampère's is not employed in practice. It is only when it is desired to determine the forces on a part of a circuit produced by the current in the remainder of the circuit, as would be the case in the simple circuit breaker illustrated in fig. 10, that difficulties arise. These are of two kinds. The first is that of the equality of action and reaction and with Ampère's formula this does not arise. According to the Biot–Savart formula the force on a current element will always be at right angles to it whether it is a complete circuit or only part of a circuit which gives rise to the force.

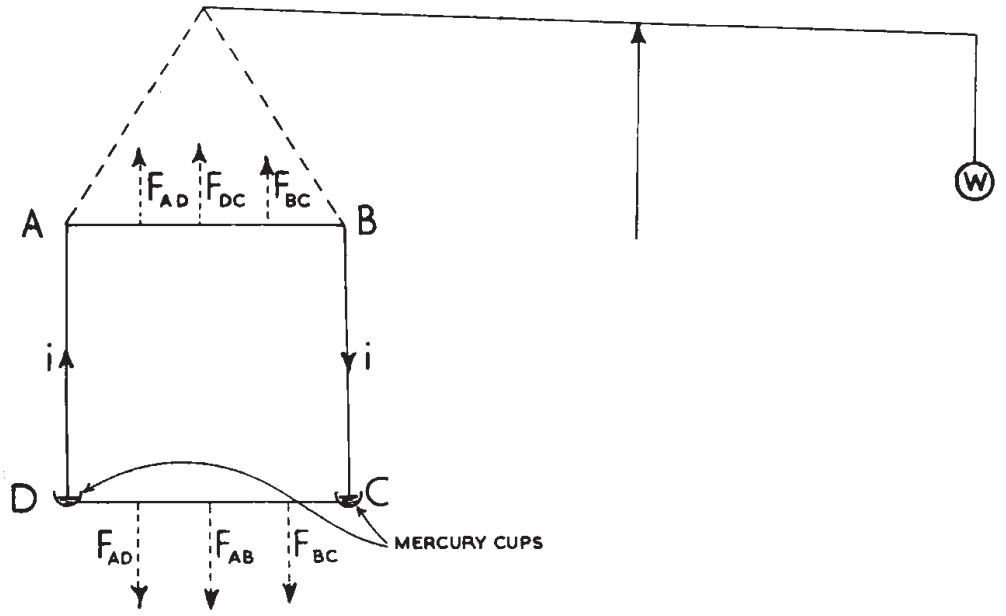

Fig. 10.

Fig. 10. An idealized circuit breaker. The square wire frame ABCD is divided by mercury cups at D and C. The upper portion DABC is counterpoised on a balance. Repulsion between it and the fixed member DC will cause it to rise when a current i is made to flow round the square (depending upon the weight of the counterpoise). Connection to the electric supply has been omitted to simplify the diagram.

If we consider according to Biot–Savart, the force on the side DC of the square circuit breaker in fig. 10 (CD is fixed but experiences a force from the remainder of the circuit which is counterbalanced by the reaction of its supports) it can be looked upon as being composed of three parts, F_{AD} due to the current in AD, F_{AB} due to the current in AB and F_{BC} due to the current in BC. If now we consider the vertical forces exerted on the other and movable part of

[1] *op. cit.*, p. 370.

the circuit DABC, they will, of course, all be exerted on the member AB, the forces on DA and BC being horizontal. Since a square circuit, if floated on a liquid, does not tend to move, the forces on the fixed and movable portions of the square must be equal and opposite. The force on AB produced by the fixed side DC, F_{DC}, will obviously, by symmetry, be equal and opposite to F_{AB}, the force exerted by AB on DC. Hence the force experienced by AB will only be equal and opposite to that experienced by CD if we include forces F_{AD} and F_{BC} caused by the sides AD and BC respectively. In other words to calculate the forces on the movable part of the square circuit it is necessary to include forces which parts of it exert on itself. If these ' self forces ' are included the Biot–Savart formula can be applied in this case also. We have, as I have said before, to invent a new mechanics, and that is not, as Rosser suggests, the mechanics of relativity.

The second difficulty applies equally to Ampère's as to the formula of Biot and Savart. It is that if the circuit be assumed to be a line then the integrals obtained in calculating the forces exerted by one part of a circuit on the remainder become infinite in both cases. The difficulty can be overcome, at least in simple cases, by a cumbersome further subdivision of the current elements into sub-elements but in practice, neither formula would be used. This difficulty is similar to that which occurs in the calculation of coefficients of self induction. It is dealt with in the standard method for calculating these coefficients and the forces required would be calculated from them rather than by employing either Ampère's or Biot and Savart's formulae.

The fact that this discussion of the relative merits of Ampère's formula and that of Biot and Savart has lasted for 150 years without reaching any fixed conclusion seems to me to illustrate the importance in the scientific field of something like a verifiability criterion of meaning. I do not wish to digress upon logical positivism or to debate the case for it in other fields than science; but in the field of science, any question to which no experimental observation can be germane—as in the case of the rival merits of the formulae of Ampère and of Biot and Savart—can clearly never be answered finally and moreover cannot affect any practical decision which has to be taken. There is, of course, a sense in which Ampère's formula is superior to that of Biot and Savart and that is that it is consistent with Newton's third law whereas Biot and Savart's is not. We are, in practice, so accustomed to the applications of Newton's laws that to work with expressions which transgress them makes our thinking very liable to error. But there is equally a sense in which the formula of Biot and Savart is superior to that of Ampère. It is simpler. We are fundamentally lazy—perhaps it would be more becoming to say that we are highly appreciative of mathematical succinctness—and choose the easiest path. But we have to be careful that we do not apply the formula out of context.

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